ESTIMATING THE PARAMETERS OF A THÉVENIN EQUIVALENT SYSTEM BASED ON OUTPUT VOLTAGE AND CURRENT MEASUREMENTS: COMPUTATIONAL CHALLENGES AND SIMULATED STUDIES

BY

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THESIS

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ABSTRACT

This thesis presents a study of the computational challenges when estimating the Thévenin equivalent parameters at a large-scale power system network based on local measurements. The estimation will use a common mathematical method for an overdetermined system. The motivation is taken from phasor measurement unit (PMU) measurements at specific nodes across a transmission line for monitoring purposes. The ac equivalent system is simplified to a small dc circuit to study the possibility of encountering an ill-conditioned measurement set. Finally a second estimation technique, based on noisy measurements, is presented and compared against the first.

To my family and beloved mother

ACKNOWLEDGMENTS

I would like to sincerely thank my adviser Peter Sauer for his tremendous support and the opportunity he gave me to be part of his research group. Thanks to my wife for the motivation she provided me throughout the development of this thesis, and finally to those who helped me with the technicalities of some of the topics; they know who they are.

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CHAPTER 1
INTRODUCTION

Nowadays, electric utilities and system operators are steadily moving the electrical grid towards an upgrade with more digital interfaces. This improved and modern grid is called the Smart Grid by industry and scholars. Part of this modernization involves the use of phasor measurement units (PMU) to obtain current, voltage and frequency information in a given transmission line. This device acquires the measurements at speeds of 10 to 60 measurements per second with a time stamp synchronized to a Global Positioning System signal. The fast and synchronized data acquisition has led to the increased use of PMU by different power grid entities, for instance, transmission system operators. One of the many applications that use PMU measurements exclusively is the estimation of a Thévenin equivalent (TE) system.

In this thesis a simplified TE system will be studied. This basic dc system will allow us to study the computational challenges when estimating the TE parameters (TEPs) using a common data fitting technique. It will also help us determine the minimum load variation needed to generate measurements for simulations. And finally, due to the noise in PMU measurements, a second estimation method will be analyzed and compared against the first.

A TE system is a reduced system representation of a larger and usually more complex power system where the output measurements of these two are electrically equivalent. The use of a TE system is broad, but for the context of this thesis, the main use will be for stability analysis.

There are different methods of estimating the parameters of a TE system. The majority is non-invasive in the sense that they depend on direct measurements, simulated equivalent systems, or any other technique, which does not affect the operation of the line or system under study. TE systems have many applications such as weak-voltage load-bus monitoring [1], voltage stability analysis [2], maximum power transfer limits [3], steady-state and transient stability, among others.

The most practical non-invasive method relies on a least square estimation (LSE) approach when PMU measurements are given. In the literature, the Thévenin equivalent parameter (TEP) estimation is referred to as *tracking* [4]. Depending on the number of local measurements or the given system, a Gauss-Newton approach can be used [4]. The proposed algorithm is used to obtain the TEP at a Canadian substation and verified using Power System Simulation for Engineering or PSS/E. In [5] a triangulation method is applied to local measurements, and care is taken to account for the drift phase among measurements, as explained in Appendix A. This method is applied to off-line PMU measurements at a wind farm to obtain its TE system for modeling and disturbances studies. Both of these methods are similar in the sense that they rely on a mathematical data-fit algorithm, namely LSE. For this reason, it is important to first study the cases for which the measurements form a well conditioned set for estimation. The theoretical analysis was reviewed in [6] for the case of estimating TEP from local measurements.

In this thesis, a practical perspective is embraced with specific simulated examples and case studies for when an ill-conditioned set of measurements is encountered. The general goal is estimation of the TEP, given a set of local PMU measurements at two different nodes of a transmission line (TL). From the discussion above and the literature in Section 1.1, we see that there is no reference to the computational limitation when using LSE. Moreover, just a few attempts have been made to estimate the parameters from noisy measurements. Thus, in this thesis we will cover these topics. Both methods and the computational challenges will be studied on a simple dc circuit derived from a large power system.

1.1 Literature Review

In the literature one technique is predominantly used to estimate the parameters of a TE system: least square. The reason, in general, is the multiple samples of measurements at hand, which this method favors, and its simplicity. However, LSE is used in different ways depending on the system under study and the available measurements. For example [2] uses a mixture of SCADA and local measurements. SCADA system provides impedance and bus topology data. In [7] local branch (instead of local bus) measurements are used. In [8]-[9] bus data information is used such as power flows and line admittances. Local PMU measurements are nowadays the preferred set of data to perform TE estimation [1]-[6]. This thesis will use the latter type of data. There are other estimation methods available, for instance: Newton-Raphson [10], weighted LSE [11], a patented method using internal PMU's information [12], state estimation techniques [13]-[14], among others [15]-[17]. This thesis will use LSE exclusively. A detailed explanation for this choice is given in Section 1.2.

Because we will be using PMU acquired data it is essential to understand the mechanism behind this technology. Appendix A explains the data acquisition process of PMU and its drawbacks. The two main drawbacks are the phase drift in each sample and the inherited noise. This latter concern can actually be used to attempt another estimation method. Noisy measurements can be used in stochastic context and a method such as maximum likelihood (ML) can be applied to estimate the TEPs.

Several papers [18]-[20] discuss ML as an estimation technique when noisy measurements are given. None of these give an application for a dc or ac power system. There exist, however, examples (with generic noise and measurements distribution) within the random process literature [21]. In all of these references the approach and algorithm are similar. The algorithm uses the notion of fitting the data to a distribution density. In the context of MLE, it looks for an argument that maximizes the likelihood of every sample to occur. Thus, depending on the nature of the distribution equations, the process gives a solution directly or by using iterative steps.

When the set of needed measurements is incomplete or has unknown signal parameters, a more complex method is used. Expectation maximization (EM) is a common technique when dealing with stochastic and noisy measurements. It offers an iterative process to find hidden signal parameters and hence acquiring indirect measurements [22]. For example, suppose measurements have the form $M = S + N$, where M is the acquired measurement, S is the signal under study and N is noise. With EM, using the samples of M, the distribution density parameters (mean and variance) of S can be found. This method is similar to ML; in fact, within the iterative algorithm, the parameters will approach those found using ML as the iterative loop reaches infinity. In this thesis a simple linear dc system is used and, even in a probabilistic context, ML satisfies our purposes. EM was discussed for completion of the methods available for the case of noisy collected data.

1.2 Motivation

In order to derive a simple dc system we will simplify a large power system to the most common circuit in electrical engineering. This technique is generally used when a simplified system is needed for specific studies. The motivation behind the use of this technique, and this paper, lies in the collected PMU data at two different nodes of a TL and the need to compute the angle difference between the TE systems on each side. Therefore, as a first step, an overview of the complete power system is presented, as well as the variables involved.

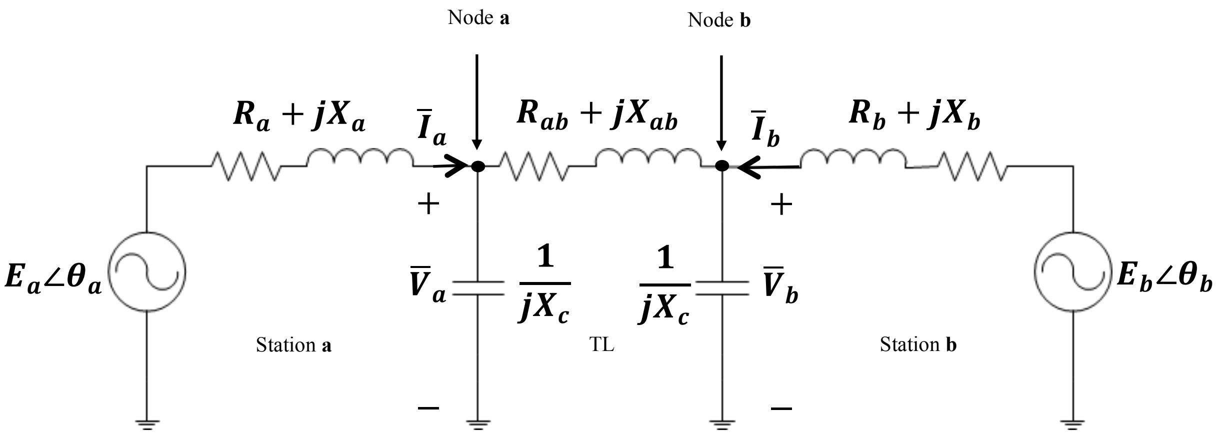

Figure 1.1 Lumped TE representations across a TL

This model, illustrated in Figure 1.1, features the most common TL components. Notice that the TL equivalent is at the center of the circuit featuring the typical parameters: impedance and shunt capacitors. The two areas at which the measurements are taken are connected to the TL at nodes **a** and **b**. This is where the PMUs are assumed to be taking the measurements. Station **a** (from the node looking to the left side) and Station **b** (from the node looking to the right side) are the TE representation we are seeking. Thus each area has a TE system in place, instead of an unknown network on it. It is important to observe that all the variables and parameters are lumped values representing a bigger system. The TEPs are unknown at this point. The only given variables are the TL parameters, the currents and voltage phasor of the PMU. These last two are given as outputs of the two defined Stations — **a** and **b**. The variable definitions are summarized below.

$\boldsymbol{R_{ab}}$ $[\Omega]$: *TL resistance. Usual value is small compared to the inductance, and hence is usually ignored*

$\boldsymbol{X_{ab}}$ $[H]$: *TL inductive reactance*

$\boldsymbol{X_c}$ $[\Omega]$: *TL reactance of shunts capacitors. These can be ignored*

$\boldsymbol{E_x}$ $[V]$: *TE lumped voltage magnitude of phasor* $\bar{E}$

$\boldsymbol{\theta_x}$ $[Radians]$: *TE voltage angle of phasor* $\bar{E}$

$\boldsymbol{\bar{V}_x}$ $[V]$: *PMU voltage measurement phasor (magnitude and angle)*

$\boldsymbol{\bar{I}_x}$ $[A]$: *PMU current measurement phasor (magnitude and angle)*

A simplified circuit will be presented next. In this circuit the shunt capacitance will be ignored, as well as the resistance of both TE systems. The result is shown in Figure 1.2.

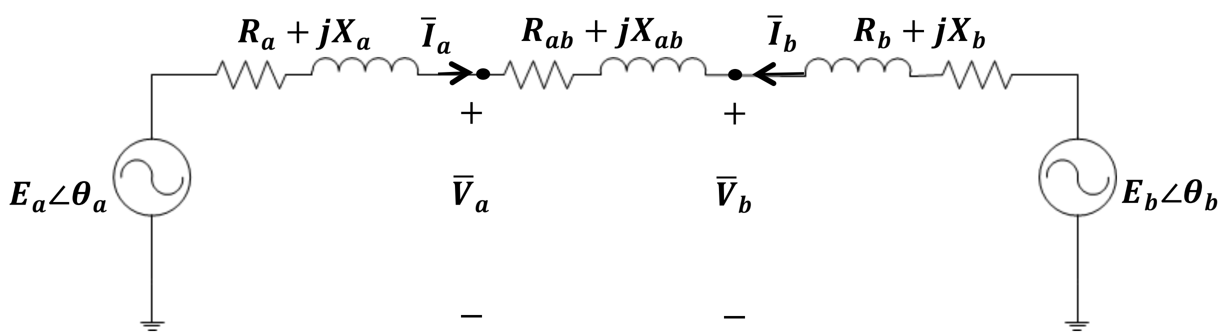

Figure 1.2 Lumped TE representations across a TL with omitted shunt capacitance

As previously mentioned, the PMUs are collecting the data at the connecting nodes **a** and **b**. This setup allows us to have voltage measurements across the capacitors. Because this information is retained independently of the capacitors' presence, and to make circuit manipulation easier, these capacitors are omitted. We now have a system that is easier to work with. We can think of it as two single machines connected through a TL, representing the TE at both ends of the TL. Notice that all impedances and TE voltages are in series. Thus we can further simplify the system and arrive at a fundamental circuit in electrical engineering. This circuit is shown in Figure 1.3.

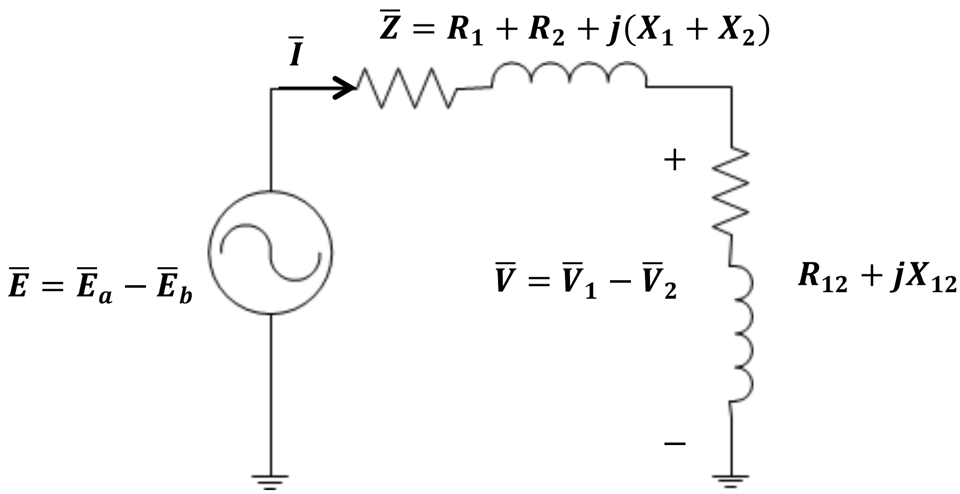

Figure 1.3 A simplified AC TE system with line impedance as load

Adding the reactances in series and taking the TE phasor voltage difference gave the circuit in Figure 1.3. The result leaves the line impedance on the right, which then behaves as a load to the system. The TEPs are the voltage source and impedance respectively. This circuit opens the possibility of perform studies on the computational limits using a load variance simulation. For this reason, the reduction from a large power system to the one in Figure 1.3 is indispensable in this thesis.

CHAPTER 2
THE PMU DATA

The PMU measurements that were collected are presented in this chapter. There are two different sets of real data examples. They differ in the number of samples per second and the acquired measurements. The first set consists of the following: line-to-line voltage phasor, frequency, 3-phase complex real power, and 3-phase complex reactive power. These measurements were taken ten times per second for a one-hour period of time. The plots in Figure 2.1-2.4 represent graphically the acquired data from Station **a**. Similar data were obtained in Station **b**, but these are not presented.

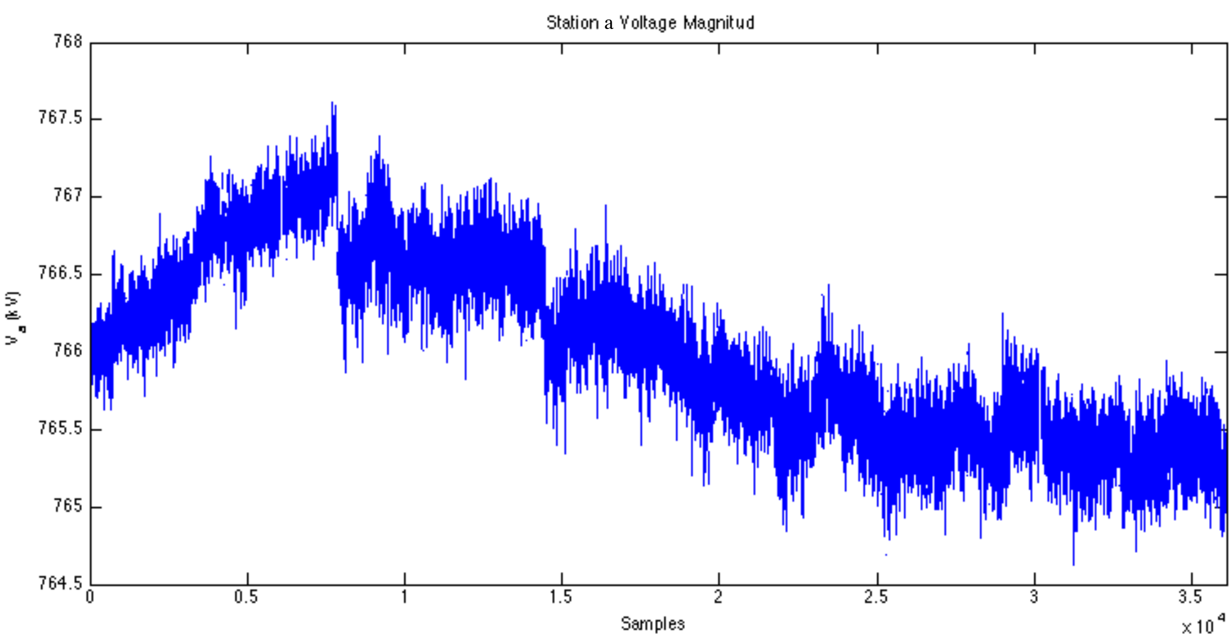

Figure 2.1 PMU measurements of voltage magnitude at Station **a**

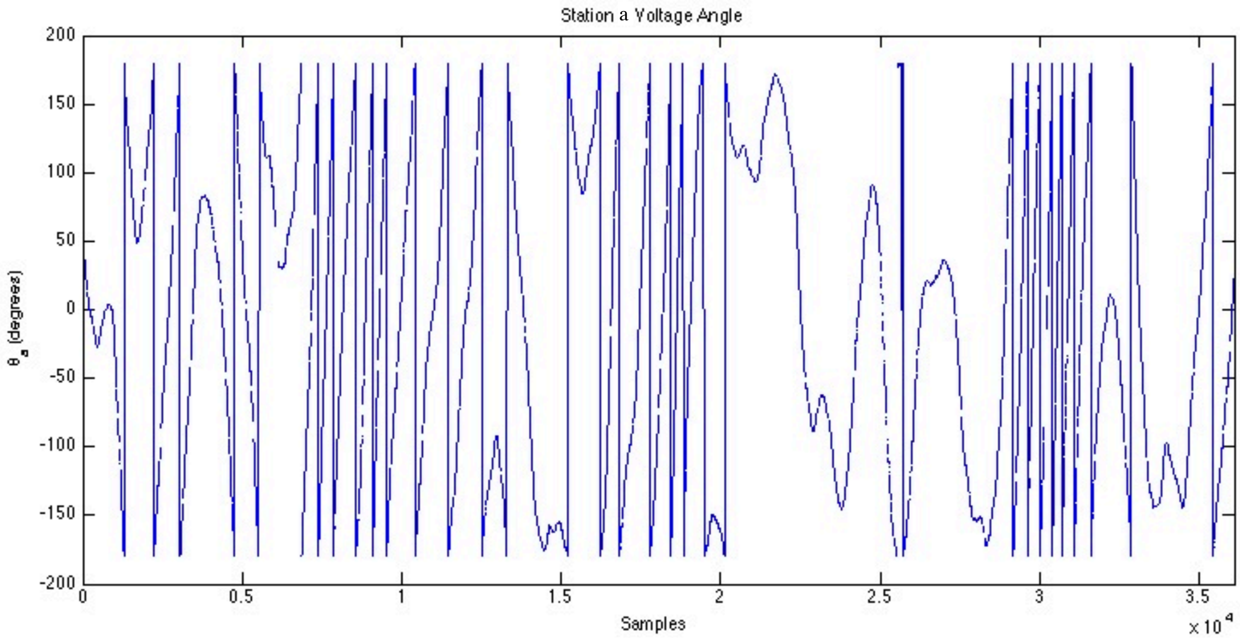

Figure 2.2 PMU measurements of the angle voltage at Station **a**

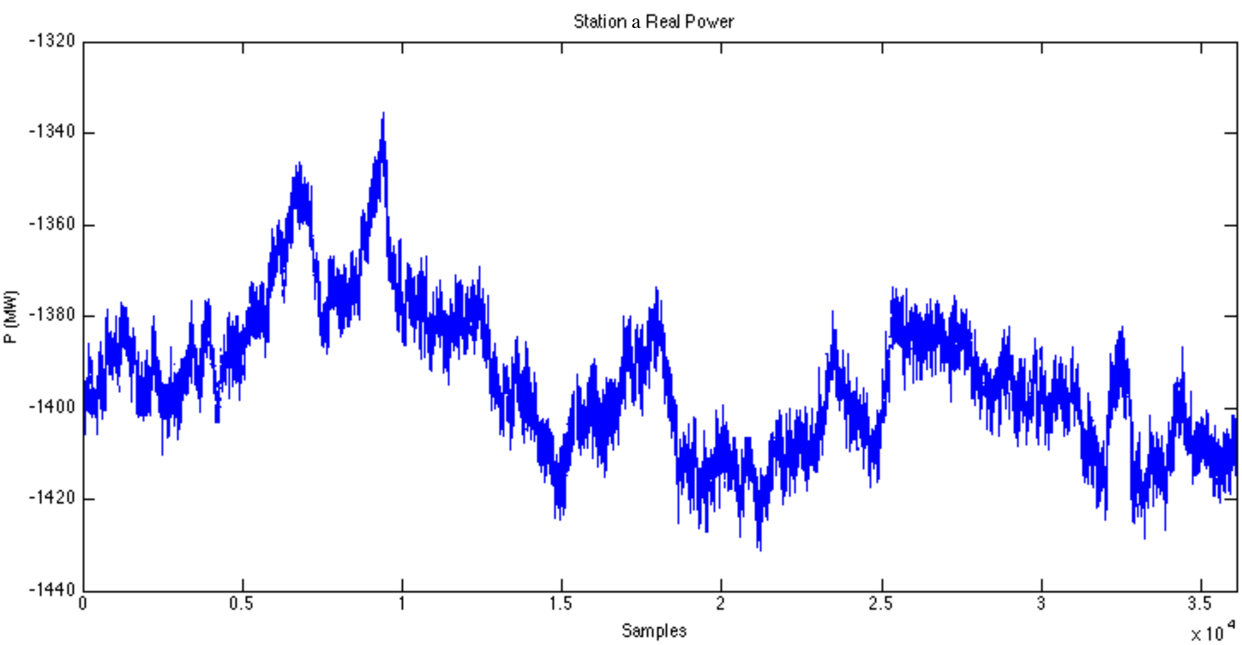

Figure 2.3 PMU measurements of real power out of Station **a**

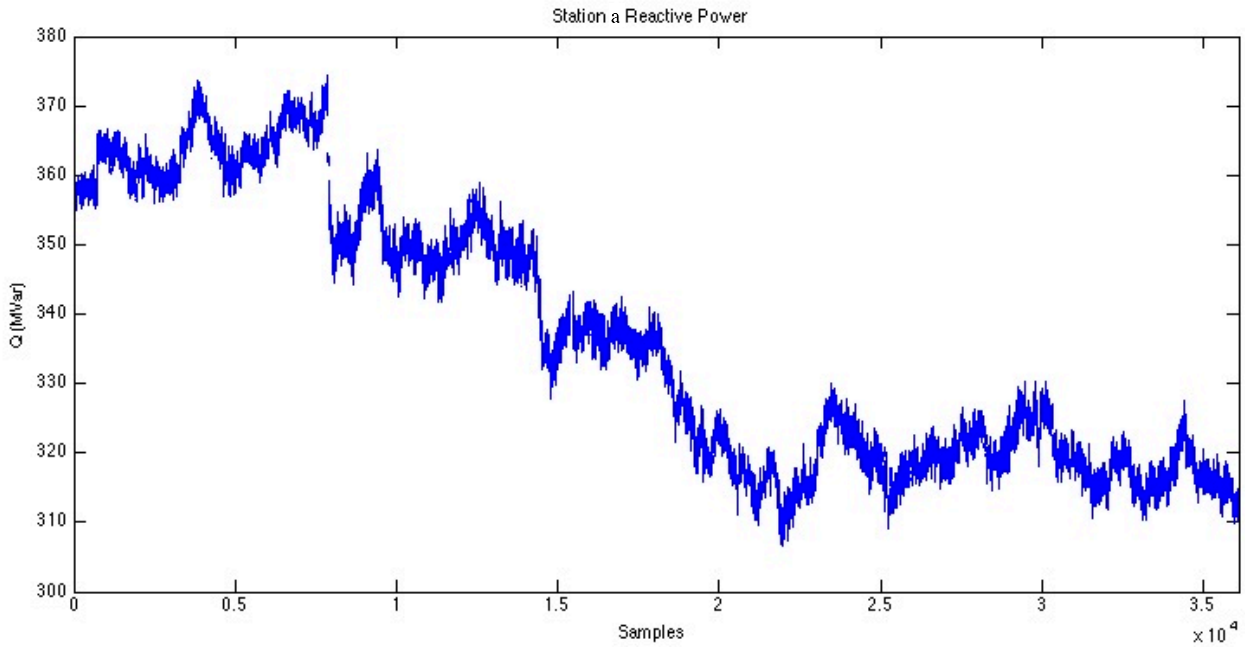

Figure 2.4 PMU measurements of reactive power out of Station **a**

It is important to point out a terminology, about the data, which is going to be used throughout this thesis. When referring to the data (PMU collected or simulated), the word measurements and samples will be used interchangeably. It is understood that measurements (voltage, current, etc.) can be seen as the collection of samples of a given signal.

2.1 Computing Missing Measurements

A simple computation from the given voltage and complex power is performed to obtain the current phasor on both ends of the TL. The operation, shown below, is applied to every measurement and then plots of the results are given.

$$\bar{S} = 3\,\bar{V}_p\,\bar{I}_p^*$$

$$\bar{S} = P + jQ$$

$$\bar{I} = \left(\frac{P + jQ}{\sqrt{3}\,\bar{V}}\right)^{*}$$

The results are thus: the given Voltage in the form $\bar{V} = |V|\angle\theta_v$ (magnitude and angle) in Figures 2.1 and 2.2, together with the computed current $\bar{I} = |I|\angle\theta_I$ (magnitude and angle) shown in Figures 2.5 – 2.6.

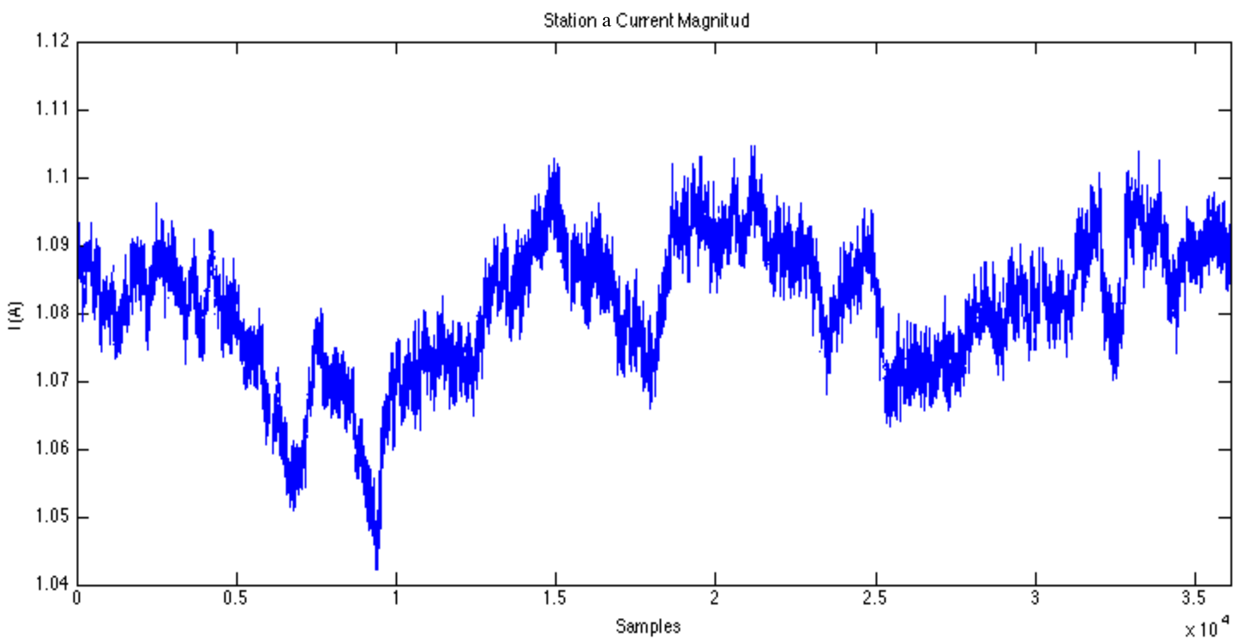

Figure 2.5 Computed measurements of the current phasor (magnitude) out of Station **a**

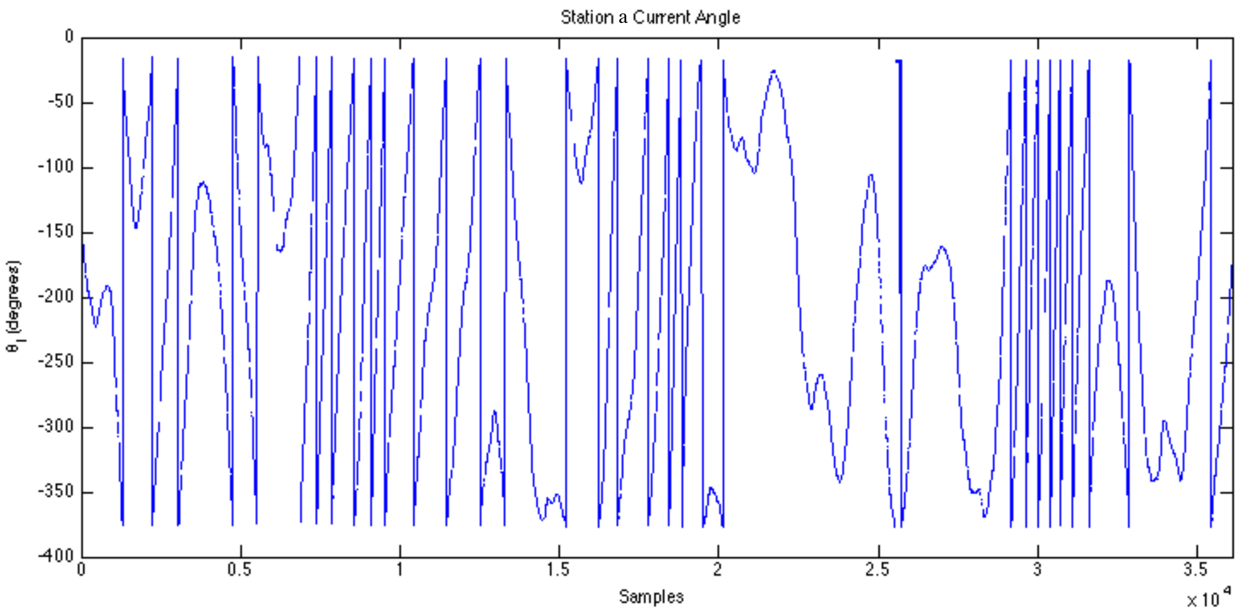

Figure 2.6 Computed measurements of the current phasor (angle) out of Station **a**

2.2 Filtering PMU Data

This set of measurements, and in general most PMU collected measurements, contains erroneous or omitted samples. For this reason some type of filtering is necessary. As an example, consider the measurement set in Table 2.1, consisting of 5 continuous samples in the same one-second period.

Table 2.1 Sample set of PMU data

Sample	Date Time	$\|V\|$	$\angle\theta_v$	f	MW	$MVar$
1	9/4/10 1:08:52 AM	766.12	0.81869	59.997	-1386.7	358.56
2	9/4/10 1:08:52 AM	766.1	0.71573	59.997	-1385.9	358.3
3	9/4/10 1:08:52 AM	NaN	NaN	NaN	NaN	NaN
4	9/4/10 1:08:52 AM	NaN	NaN	NaN	NaN	NaN
5	9/4/10 1:08:52 AM	765.95	0.43306	59.998	-1391.1	357.39

There exist a few strategies that can be taken to fix this discrepancy. These are discussed below:

Filter erroneous data: The most convenient and easiest way to handle this issue will be to overlook or remove the erroneous samples from the computation algorithm. They are recognized by the notification "NaN", which occurs when the PMU fails to store the sample or an internal computation yielded a non-numerical value. Since ten samples per second are taken, a few other measurements will remain to work with. The sample set will be reduced and care has to be taken when using LSE. In the case of 8 or more missed measurements the whole data set might have to be skipped or a different filtering method will need to be used.

Average sample data set: An alternative is to simply compute the average of the sample set. This means to take an average of the ten measurements every second. This is also a simple option although it might need more computation.

Interpolation: A third option is to interpolate the missed data using the previous and next available measurements. This option tends to be challenging when more than two consecutive measurements are missing from the set.

Low-Pass filter: This filter is used in situations where the measurements do not have a smooth curve representation or have noticeable noise. In such cases the measurements are passed through a digital filter that will output a set with a smoother plot.

In this thesis we will be using both the first and second strategies within the LSE algorithm. An example code is presented in Appendix B.

2.3 A Second Set of Measurements

The second set will mainly be used for comparison and demonstration of some implications regarding the LSE algorithm. It consists of PMU data collected at a different TL in a period of 24 hours with 32 samples per second. Every sample provides the voltage phasor and current phasor. Hence, no extra computation is required. This set has no missing or erroneous data. It is assumed that a previous filtering of some kind has taken place, leaving us with a clean set of PMU measurements. The data is shown in Figures 2.7 – 2.8.

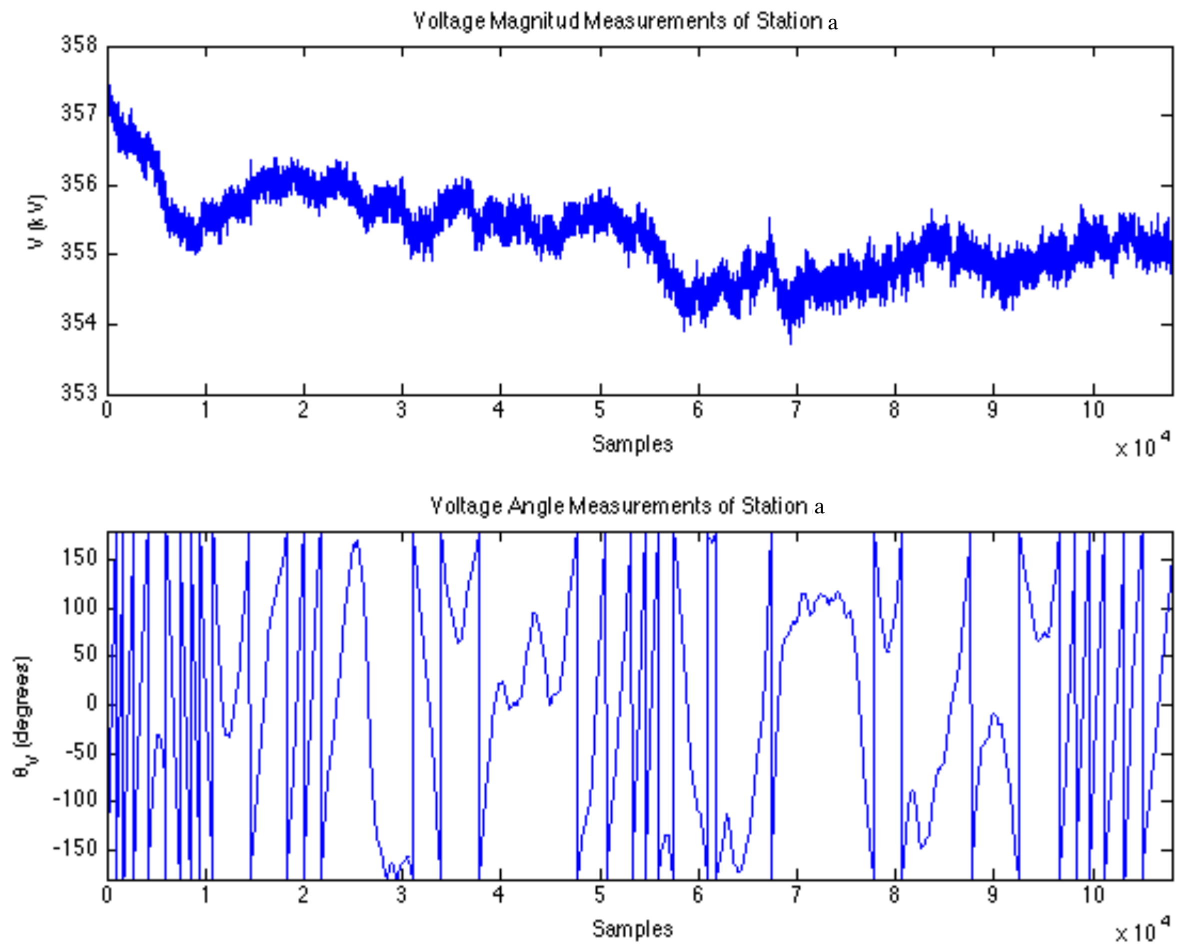

Figure 2.7 Second set of PMU voltage phasor measurements (magnitude and angle)

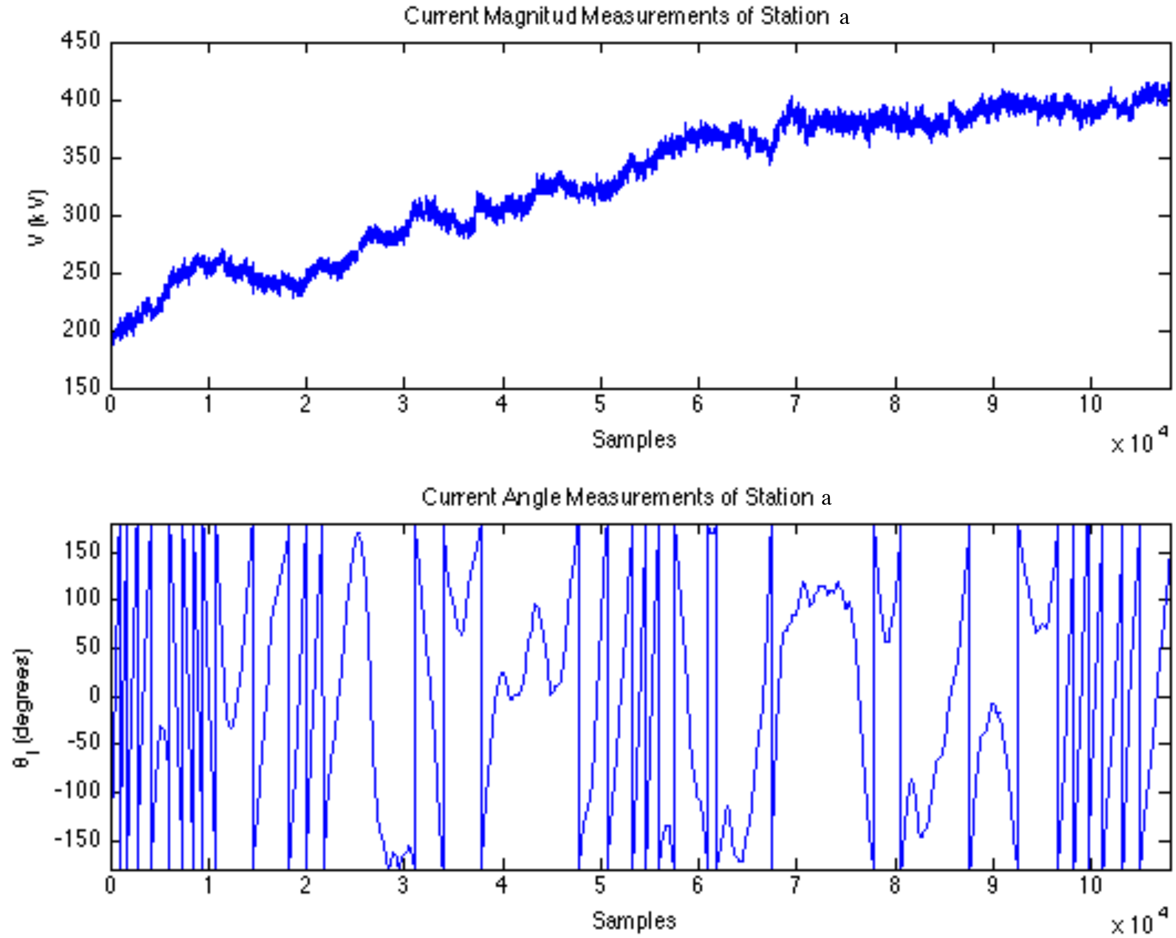

Figure 2.8 Second set of PMU current phasor measurements (magnitude and angle)

CHAPTER 3

THE DC SYSTEM MODEL

It is essential to study the methods available for estimation on a simple dc circuit before we attempt to develop an algorithm to estimate the TEP on the system in Figure 1.3. In this manner quick simulations can be performed and the technicality of the methods can be studied. At the same time the computational challenges can also be explored. The equivalent dc system, shown in Figure 3.1, is similar to the one in Figure 1.3. As the circuit no longer involves phasors the analysis is simplified. However, the conclusions also apply to the ac circuit.

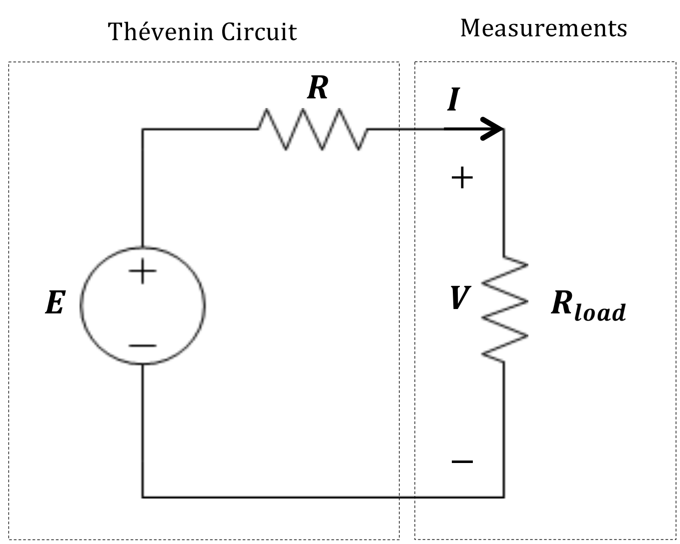

Figure 3.1 A dc circuit representation of the system in Figure 1.3

The equations governing this circuit are:

$$KVL: \;\; E = \boldsymbol{V} + \boldsymbol{I}R \tag{3.1}$$

$$VDR: \;\; \boldsymbol{V} = \frac{R_{load}}{R + R_{load}} E \tag{3.2}$$

$$Ohm's\ Law: \quad \boldsymbol{I} = \frac{V}{R_{load}} = \frac{1}{R + R_{load}} E \tag{3.3}$$

where

- E (V) is the TE voltage
- R (Ω) is the TE resistance
- $R_{load}(\Omega)$ is the resistive load
- V (V) is the voltage measurement
- I (A) is the current measurement.

The problem in general is stated as follows:

Given V and I as sets of measurements, i.e., given pairs (V_i, I_i) for $i = 1,2 \ldots n$; compute the TEP E and R.

From Equation (3.1) governing the circuit above we can see that we have three different cases:

- For the case $n = 1$

  The system has no solution because there is only one equation with two unknowns E and R. This is known as an *underdetermined* system.

- For the case $n = 2$

  The system has a unique solution because in this case we have two equations and two unknowns. This is true provided the two measurements (V_i, I_i) $i = 1,2$, do not make the two equations linearly dependent.

- For the case $n > 2$

  Here we will have an $overdetermined$ system, since there will be more equations than unknowns. An estimation technique must be applied in order to compute E and R.

3.1 Least Square Estimation (LSE)

From now on it will be best to define the measurements in a vector form as follows:

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ .. \\ V_n \end{bmatrix}, \ \boldsymbol{V} \in \mathbb{R}^n \quad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ .. \\ I_n \end{bmatrix}, \ \boldsymbol{I} \in \mathbb{R}^n \quad n \geq 2 \tag{3.4}$$

And, as previously stated, the measurements are in groups of ten samples per second. For a one-hour period this gives us a total of 36000 samples. Hence $n = 10$ for every one-second subset of measurements. Given the overdetermined nature of the relation in Equation (3.1) it is practical to use a data fit technique to obtain the TEPs that reduce the error between the estimated and actual measurements. In short, given an estimated value for the TEP, i.e., $\hat{E}$ and $\hat{R}$, we want

$$\hat{E} - \left(V_1 + I_1\hat{R}\right) \approx 0$$

More precisely, for $n\ = 10$, we want to minimize L, for $\hat{E}$ and $\hat{R}$, where

$$L \triangleq\ \hat{E} - \sum_{n=1}^{10} V_n - \hat{R}\sum_{n=1}^{10} I_n$$

or in vector form:

$$\begin{bmatrix}\hat{E}\\\hat{R}\end{bmatrix} = \underset{E,R}{\operatorname{argmin}}\left([1\ \ -\boldsymbol{I}]\begin{bmatrix}E\\R\end{bmatrix} - \boldsymbol{V}\right)$$

Define $A \triangleq [1\ \ -\boldsymbol{I}], x \triangleq \begin{bmatrix}E\\R\end{bmatrix}$ and $b \triangleq \boldsymbol{V}$, where $\boldsymbol{V}, \boldsymbol{I} \in\ \mathbb{R}^n$; then we effectively want to minimize

$$L \triangleq Ax - b$$

and thus

$$\hat{x} = \underset{x}{\operatorname{argmin}}(Ax - b)$$

The form of this relation is a common mathematical expression in linear algebra. Moreover, given that this linear system is overdetermined, the preference of LSE follows intuitively. The system of equations has the following form:

$$\underline{A}\,\underline{x} = \underline{b}$$

$$\underline{A} \in \mathbb{R}^{m\times n}, \qquad \underline{x} \in \mathbb{R}^n, \qquad \underline{b} \in \mathbb{R}^m$$

Here $\underline{x}$ is the unknown parameter. We are looking for solutions in the case $m > n$. The solution sought is such that it minimizes $\|\underline{A}\,\underline{x} - \underline{b}\|$. That is, it minimizes the error, or residues, between measurements. This procedure effectively makes the data fit on a linear approximation. We can go further and minimize the square of the errors as follows:

$$f(x) = \frac{1}{2}\|\underline{A}\,\underline{x} - \underline{b}\|^2$$

To find the minimum of this function, we take its derivative with respect to $\underline{x}$. We know that at its optimum, the solution $\hat{x}$, will make the derivative of $f(x)$ vanish.

$$\frac{\partial f(x)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2}\|\underline{A}\,\underline{x} - \underline{b}\|^2\right)_{\hat{x}} = 0$$

$$= \frac{\partial}{\partial x}\left(\frac{1}{2}\left(\underline{x}^T\underline{A}^T\underline{A}\,\underline{x} - 2\underline{x}^T\underline{A}^T\underline{b} + \underline{b}^T\underline{b}\right)\right)_{\hat{x}}$$

$$= \underline{A}^T\underline{A}\,\underline{\hat{x}} - \underline{A}^T\underline{b} = 0$$

$$\underline{\hat{x}} = \left(\underline{A}^T\underline{A}\right)^{-1}\underline{A}^T\underline{b} \tag{3.5}$$

It has been proven that (3.5) gives a unique least-square solution [23] for the estimated $\hat{x}$, provided that $\underline{A}$ is full rank (measurements are linearly independent).

3.2 Real Case Application of LSE

For this application we will use the data magnitudes, given in the second set of the data in Chapter 2 for system A. The magnitude of $\boldsymbol{I}$ is given in Figure 2.6. The magnitude of V is given in Figure 2.7. We can use LSE as explained above, and estimate the TEP every second. Following the LSE algorithm, the TEPs are given in Figures 3.2 and 3.3.

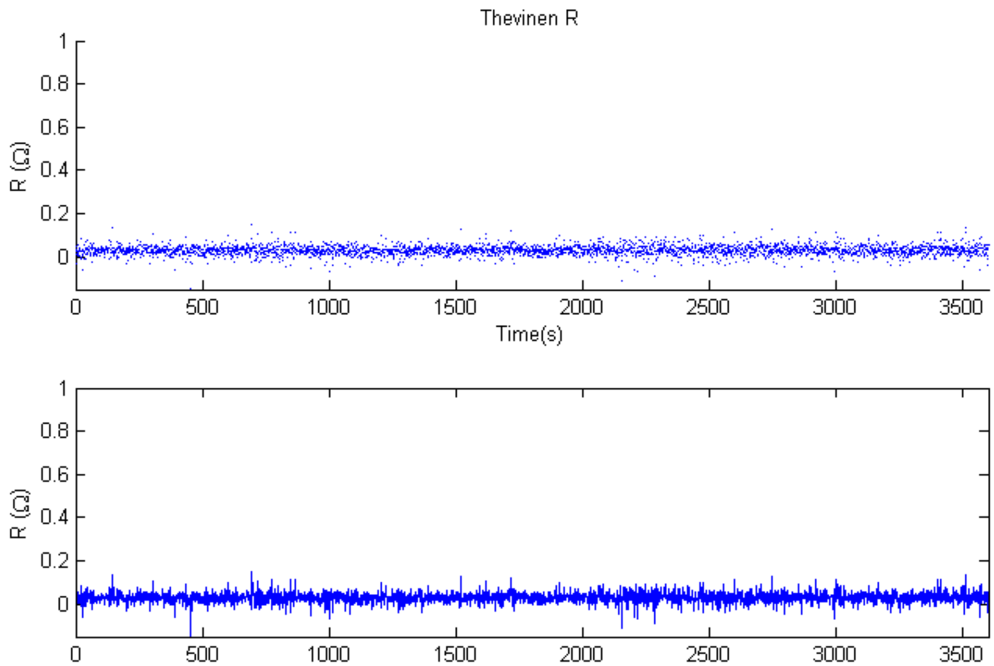

Figure 3.2 LSE result for R. Scatter plot (top) and continuous plot (bottom).

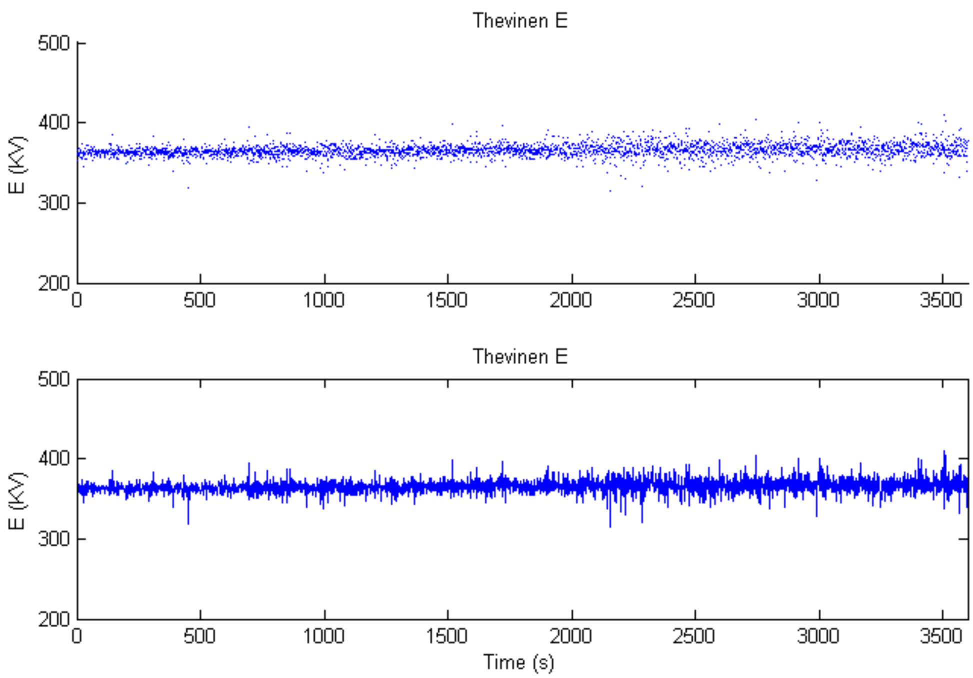

Figure 3.3 LSE result for E. Scatter plot (top) and continuous plot (bottom).

In Figures 3.2 and 3.3, it is important to observe that at every second we make the assumption that E and R are constant over the period of estimation, i.e., for one second. This assumption is applied throughout the 24-hour time frame under study. In the plots we see the TEPs oscillate around a mean value over time, but they change value at every second. For E, the mean value is 376 kV and for R it is $0.12\ \Omega$. The change is small and will depend on the load variation. This variation is more pronounced towards the end.

3.3 Generating Simulated Measurements for the Studies of Ill-Conditioned Cases

Least-square estimation becomes the easiest and most effective method to estimate any unknown parameters when many measurements exist. However, ill-conditioned matrices could result from a set of given measurements. We will next study the different ways an ill-condition system can occur. For the following studies, the voltages will be assumed to be in per unit values (fraction of the nominal or base value), the value of E_a and E_b have to be around 1 p.u. each. Thus, the analysis will be performed on small differences such that $E_1 - E_2 \approx 0.1\ \mathrm{p.u.}$ The resistance of distribution transmission lines is usually very small and hence all resistances will have a value close to zero or $R_{all} \approx 0.1\ \Omega$.

3.4 Study 1 (E, R Constants)

Four different scenarios will be studied. In each of them one parameter will be varied at the time. The parameters are: resistive load, resistance, and voltage source. For this first study we assume the voltage source and the resistor are held constant and the resistive load varies. We will use the circuit in Figure 3.1 as a reference and Equations $3.1 - 3.3$.

The measurements will be the current ($\boldsymbol{I}$) and voltage ($\boldsymbol{V}$). They will change if R_{load} varies (given that E and R remain constant) or with noise inherited in the measuring instrument (this will be discussed in Chapter 4). R_{load} can change for different reasons. The most common is temperature. The change follows the approximation

$$R_{load}(T) = R_{load}(1 + \alpha\,(T - T_o))$$

where T is temperature and α is the temperature coefficient of resistance in the 10^{-3} range. (The same idea follows in a real TL, but in that case we also have of course load variation.) A unique solution for E and R can be found, using Equation (3.1), as long as we have at least two sets of linearly independent measurements — I and V. If we had only one set of measurements, then Equation (3.1) will have infinitely many solutions. Using the values specified in Section 3.3, and Equations (3.2) − (3.3) we generate the first set of data:

$$V = \frac{0.1}{0.2}0.1 = 0.05 \qquad I = \frac{0.05}{0.1} = 0.5$$

$$E = 0.05 + 0.5\,R \quad (First\ set)$$

For the second data set, we would like to obtain the smallest change of R_{load}, which would not produce an ill conditioned system. In other words, we look for an ϵ such that $R^{*}_{load} = R_{load}(1+\epsilon) = 0.1(1+\epsilon)$ gives us a set of measurements independent of the first set. This, in turn, will allow us to compute the parameters from the system of equation formed by sets 1 and 2.

$$E = \frac{(1+\epsilon)}{(2+\epsilon)}0.1 + \frac{1}{(2+\epsilon)}\,R \quad (Second\ set)$$

If we were to solve this system of equations manually, we would more likely use a direct approach, such as substitution. Symbolically, the value of ϵ can be as small as we wish; numerically, however, computations become harder as $\epsilon \to 0$. The ability to solve the system for very small values of ϵ will depend on the word length of the calculator or the software being used. For our purpose, MATLAB will be used to perform this calculation. Since the measurements will usually be given in an ordered array, we input the system in MATLAB in the following format:

$$\begin{bmatrix} 1 & -0.5 \\ 1 & -\dfrac{1}{(2+\epsilon)} \end{bmatrix} \begin{bmatrix} E \\ R \end{bmatrix} = \begin{bmatrix} 0.05 \\ \dfrac{(1+\epsilon)}{(2+\epsilon)} 0.1 \end{bmatrix} \tag{3.6}$$

For a 32-bit version of MATLAB, the smallest value epsilon can reach is $\boldsymbol{\epsilon = 1x10^{-14}}$ before the linear system becomes singular. This means, at least computationally, a second set of measurements will be useful as long as the new R^*_{load} increases by at least 10^{-12} %. An increment higher than this ensures MATLAB will find a unique solution for the system above. An increment below ϵ will make the system ill conditioned, and the parameters simply cannot be computed by any means. It will effectively leave us with a system with a single equation and two unknowns. To find the value of ϵ, a trial and error method was used and the code can be seen in Appendix B.

Real measurements, such as those obtained in a lab, are unlikely to have a variation of the size of ϵ (because of noisy or load change). This is mainly due to the limited resolution of the instruments. However even with a limited-resolution instrument, we can still see a real variation in the load resistance from one measurement to the next. For example an increase of load resistance by 10% - 20% or above will create a change. For completion of this study we can simulate an increase on R_{load} of 10% or more. By following the previous steps of generating sets 1 and 2, we arrive to the following result (with $R^*_{load} = 0.1R_{load}$):

$$\begin{bmatrix} E \\ R \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ 1 & -\dfrac{1}{(2.1)} \end{bmatrix}^{-1} \begin{bmatrix} 0.05 \\ \dfrac{(1.1)}{(2.1)} 0.1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

Given that this is a linear system, it does not matter how R_{load} changes. The changes in the rest of the variables will follow accordingly, and the exact solution for (3.1) can always be found. The reason for this is given in the setup of the system and the matrix in Equation (3.6) where we see these two equations intercept at (0.1,0.1), regardless of R_{load} since

$$\frac{(1+\epsilon)}{(2+\epsilon)}0.1+\frac{1}{(2+\epsilon)}\ R=0.05+0.5R$$

$$R(-0.5\epsilon)=0.05(2+\epsilon)-0.1(1+\epsilon)=-0.05\epsilon$$

$$R=0.1$$

Hence $E=0.1$. This means the next generated line set (Second set) will pass through the same (E,R) point for any $\epsilon>0$. The graph, in Figure 3.4, summarizes this finding.

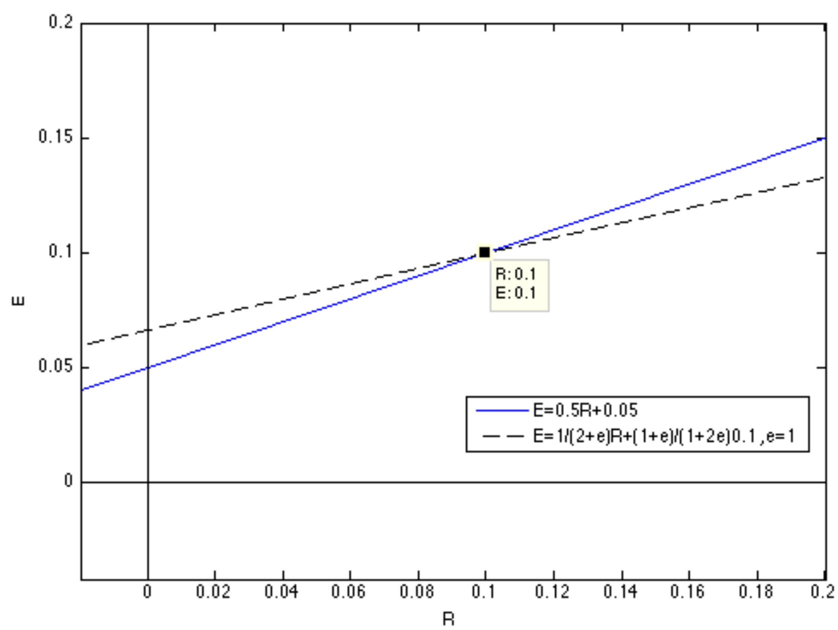

Figure 3.4 Linear dependency of generated measurements

However, when we have noise in the measurements, even a fixed R_{load} will produce different values of V, I. Its randomness will be spread on the measurements and a variation will be added to each sample. This noise will follow a density distribution, such as Gaussian. In such a case, as well as in overdetermined systems, an estimator needs to be used.

3.5 Study 2 (R,R_{load} Constants)

For this study some previous conclusions will be used for consistency and compactness. The variation of E on real scenarios is due to temperature as well as the system behavior. Every new set of measurements will be generated using $E^* = E(1+\epsilon)$. The system of equations from $(3.1) - (3.3)$ and the given values becomes:

$$E = 0.05 + 0.5\,R$$

$$E = 0.05\,(1+\epsilon) + 0.5(1+\epsilon)R \quad for\ \epsilon > 0$$

When solving for R we get

$$R\big(0.5 - 0.5(1+\epsilon)\big) = -0.05\big(1 - (1+\epsilon)\big)$$

$$R = \frac{0.05\epsilon}{-0.5\epsilon} = -0.1$$

And thus $E = 0$. This is not the expected solution. We got this solution because when setting up the system of equations, we assumed E and R were constants. By changing E to get a second set of measurements, we are contradicting the initial assumption. Thus we get an unrealistic solution. Hence, we cannot vary E for the purpose of generating measurements to compute E and R.

3.6 Study 3 (E,R_{load} Constants)

This study is very similar to the first one. The sets of generated measurements will now have the form (with $R^* = R(1+\epsilon)$):

$$E = 0.05 + 0.5\,R$$

$$E = \frac{1}{(2+\epsilon)}0.1 + \frac{1}{(2+\epsilon)}\,R \quad for\,\epsilon > 0$$

In matrix form, the above system becomes:

$$\begin{bmatrix} 1 & -0.5 \\ 1 & -\dfrac{1}{(2+\epsilon)} \end{bmatrix} \begin{bmatrix} E \\ R \end{bmatrix} = \begin{bmatrix} 0.05 \\ \dfrac{1}{(2+\epsilon)}0.1 \end{bmatrix}$$

When solving the system, we again get $R = -0.1$ and $E = 0$.

Therefore, we can never solve (3.1) by generating measurements in this way. As in the previous study, this contradicts the assumption that E and R are fixed, while the rest of the system varies to generate (V, I) pairs whether generated or measured. A counterargument to this conclusion can be that the two resistances are in series and therefore can be switched. Then Study 1 can be repeated. However, the pair (V, I) will not be the same in this re-arranged system. We are using the voltage across R_{load} for our calculations and not the voltage across R. If the right voltage (that is across R) were used, then this study would have the same results as Study 1 with the change of R_{load} being R for all the calculations.

3.7 Study 4 (All Parameters Vary)

In this case we will vary all of the values to generate the second set of measurements, i.e., $E, R, R_{load} \to E^{new}, R^{new}, R_{load}^{new}$. From previous analysis we expect such system will contradict our assumption. The second set has the following relation: $E = 0.5(1+\epsilon) + 0.5R$. And the complete system becomes:

$$E = 0.05 + 0.5R$$

$$E = 0.05(1+\epsilon) + 0.5R$$

It can be seen that this system has no solution. The two equations are two parallel lines on the E-R plane. Moreover, the rank of the matrix form is

$$rank\left(\begin{bmatrix} 1 & -0.5 \\ 1 & -0.5 \end{bmatrix}\right) = 1$$

and the system becomes ill-conditioned. This further proves the fact that we can vary only the load resistance to generate more sets to compute system parameters.

3.8 More Than Two Measurements

It was ensured in the previous studies that in order to solve for the system in Equation (3.1) we need at least two samples of $\boldsymbol{V}, \boldsymbol{I} \in \mathbb{R}^{\boldsymbol{n}}$ where $n \geq 2$ to solve for E and R. We already studied the case for $n = 2$ and saw the minimum conditions of variation R_{load} must have in order to achieve a unique solution. For the case $n > 2$, Equation (3.1) becomes an overdetermined system as we initially pointed out. An estimator, such as the LSE, has to be used. The samples will still need to obey the $\epsilon \geq 10^{-14}$ variation rule in order to avoid a singular system.

CHAPTER 4
ESTIMATION WITH NOISY MEASUREMENTS

This section explores a second estimation technique. Maximum likelihood estimation (MLE) is used in the case of many noisy measurements. This will give us two different methods of estimating the parameters of a Thévenin equivalent dc system for a set of measured data from a dc circuit with unknown parameters. As we saw, the goal is to find the solution of an overdetermined system given a set of data, in this case, noisy data. The measured or observed data are the output current and voltage of the dc circuit described in Figure 3.1. The following assumptions are made:

- The samples (observations, in this probabilistic context), current and voltage measurements, are considered “noisy” and follow a Gaussian distribution with known mean and variance.
- These observations are linearly independent, i.e., there are no duplicates in the output data.
- The actual topology of the power system is unknown; the output measurements are used to get an equivalent system, in this case a Thévenin.

MLE can be used as an estimation technique for random processes and, in this case, for processes involving noise. Assuming a Gaussian distribution of the given noise, the given samples are fit to the Gaussian density equation involving the unknown variables. For example, assuming the voltage measurements take the form

$$\boldsymbol{V} \sim N(\mu, \sigma) \quad (4.1)$$

where $\boldsymbol{V}$ is a Gaussian distribution around a mean μ and a small variance σ, then a plot of the samples will be similar to the one in Figure 4.1.

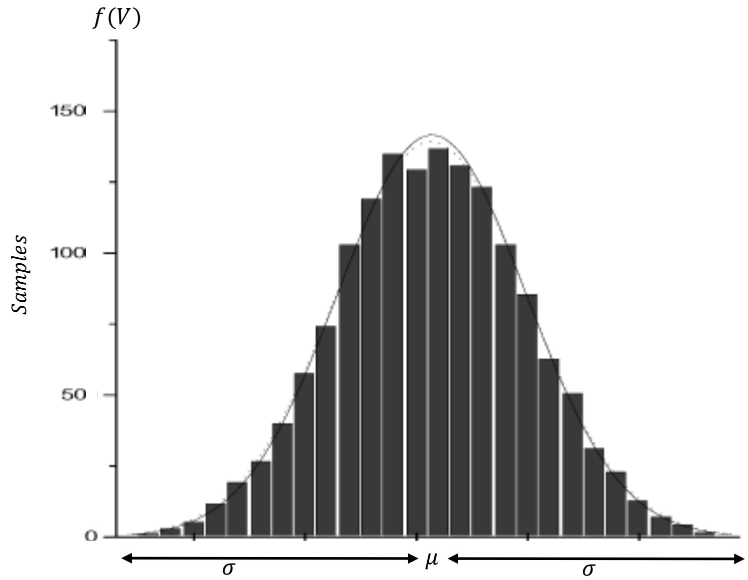

Figure 4.1 Gaussian distribution sample sets of $\boldsymbol{V}$ and the fit done by ML

Given a set of samples of the distribution of V, ML will look for the best argument of the relation $f(V)$ that maximizes the likelihood of each samples or observation will occur. In our case, ML will find the value of μ that maximizes $f(V)$. Given the linear relation of V to E and R, μ will be a linear function of these variables as well. Thus by applying ML, we are effectively looking for

$$\underset{\mu,\sigma}{\operatorname{argmax}}\big(f(V)\big)$$

V was defined in (4.1) and the maximizing arguments are a function of the TEPs

$$\mu, \sigma \sim g(E, R)$$

4.1 Generating Simulated Data

For this study, data was generated using MATLAB random function for noise simulation, with the assumption that:

$$R \sim N\ (R_o, 1),\ E \sim N\ (E_o, 1) \text{ and } \boldsymbol{I} \sim N\ (\mu, 2)$$

That is, R, E and $\boldsymbol{I}$ are Gaussian distributions around some mean. By using Equation (3.1) voltage samples can be generated. The code in Appendix B shows how 30 samples of measurement were generated. It is understood that n samples can be generated using the same code. Thus, in general the observed data is given as

$$\boldsymbol{I} \in \mathbb{R}^n \quad \boldsymbol{V} \in \mathbb{R}^n \tag{4.2}$$

4.2 LSE Revisited for Comparison

For reference, Equation (3.1) is modified for the context of this chapter as follows:

$$\boldsymbol{V} = \hat{e} - \boldsymbol{I}\hat{r} \tag{4.3}$$

where $\boldsymbol{V}$ and $\boldsymbol{I}$ are column vectors specified in (4.2) and represent the observed data for n measurements. The changes $E = \hat{e}$ and $R = \hat{r}$ were made because these are constants and the parameters to be approximated. As we saw in Chapter 3 the relation above can be written in the following matrix form:

$$\boldsymbol{V} = [\mathbf{1} \quad -\boldsymbol{I}] \begin{bmatrix} \hat{e} \\ \hat{r} \end{bmatrix}$$

If we let $\boldsymbol{M} \triangleq [\mathbf{1} \quad -\boldsymbol{I}]$ we can estimate $\hat{e}$ and $\hat{r}$ as follows:

$$\begin{bmatrix} \hat{e} \\ \hat{r} \end{bmatrix} = (\boldsymbol{M}^t\boldsymbol{M})^{-1}\boldsymbol{M}^t\boldsymbol{V}$$

The estimated $\hat{e}, \hat{r}$, according to our results, get better as the number of samples increase (Table 4.1 on page 38). The MATLAB code to compute the estimated parameters is given in Appendix B. It uses the generated data in Section 4.1.

4.3 Maximum Likelihood Estimation (MLE)

The maximum likelihood approach takes in consideration the current and voltage noise in the measurements, taking the form:

$$\boldsymbol{V} = \boldsymbol{V_o} + \boldsymbol{N}_1 \text{ and } \boldsymbol{I} = \boldsymbol{I_o} + \boldsymbol{N}_2$$

here V_o and I_o are the current and voltage mean value respectively. N_1 and N_2 are the noise in each sample. These noises are assumed to be Gaussian distributed with 0 mean and some variance $\left(N(0,\sigma)\right)$. This allows us to assume a Gaussian distribution for the observed current data:

$$\boldsymbol{I} \sim N(\mu, \sigma^2)$$

and the density function for the current is given by

$$f(I) = \frac{1}{\sigma\sqrt{2\,\pi}} e^{-\frac{1}{2}\left(\frac{I-\mu}{\sigma}\right)^2}$$

By using Equation (4.3) we can derive the conditional density function of the voltage $\boldsymbol{V}$, given the constant parameters $\hat{e}$ and $\hat{r}$. By defining the vector $\boldsymbol{\theta} \triangleq [\hat{e} \quad \hat{r}]'$ we obtain

$$\boldsymbol{V} \sim N(\hat{e} - \mu\hat{r}, \hat{r}^2\sigma^2)$$

$$f(v|\theta) = \frac{1}{\sigma\hat{r}\sqrt{2\,\pi}} e^{-\frac{1}{2}\left(\frac{v-(\hat{e}-\mu\hat{r})}{\sigma\hat{r}}\right)^2}$$

where $v \in \boldsymbol{V}$ is a single sample of the observed data. This function is known as the likelihood function and it determines the likelihood of event v occurring given θ. The total probability (using all the samples) is the product of all the individual sample probabilities. In mathematical notation this means

$$P(\boldsymbol{V}|\boldsymbol{\theta}) = \prod_{i=1}^{n} f_i(v_i|\theta)$$

$$P(\boldsymbol{V}|\boldsymbol{\theta}) = \prod_{i=1}^{n} \left(\frac{1}{\sigma\hat{r}\sqrt{2\,\pi}} e^{-\frac{1}{2}\left(\frac{v_i-(\hat{e}-\mu\hat{r})}{\sigma\hat{r}}\right)^2} \right) \tag{4.4}$$

The objective is to find a $\boldsymbol{\theta} \triangleq [\hat{e} \quad \hat{r}]'$ that maximizes Equation (4.4). This is the main purpose of ML and it guarantees a θ which causes event $v_i \in \boldsymbol{V}$ be more likely to occur. To find a $\boldsymbol{\theta}$ that maximizes (4.4) we proceed as follows:

First take the log of the likelihood function. The result is

$$\log\big(P(\boldsymbol{V}|\boldsymbol{\theta})\big) = \sum_{i=1}^{n} \left(-\frac{\log(2\pi)}{2} - \log(\sigma\hat{r}) - \frac{1}{2}\left(\frac{v_i - \hat{e} + \mu\hat{r}}{\sigma\hat{r}}\right)^2 \right)$$

$$\log\big(P(\boldsymbol{V}|\boldsymbol{\theta})\big) = -n\frac{\log(2\pi)}{2} - n\log(\sigma\hat{r}) - \frac{1}{2\sigma^2\hat{r}^2}\sum_{i=1}^{n}(v_i - \hat{e} + \mu\hat{r})^2 \tag{4.5}$$

Then take the derivative of Equation (4.5) as follows

$$\frac{d\big(\log\big(P(\boldsymbol{V}|\boldsymbol{\theta})\big)\big)}{d\theta} = 0$$

Splitting the derivative into two partial derivatives we get

$$\frac{\partial\big(\log\big(P(\boldsymbol{V}|\boldsymbol{\theta})\big)\big)}{\partial\hat{e}} = \frac{1}{\sigma^2\hat{r}^2}\left(\sum_{i=1}^{n}(v_i - \hat{e} + \mu\hat{r})\right) = \frac{1}{\sigma^2\hat{r}^2}\left(\sum_{i=1}^{n} v_i - n\hat{e} + n\mu\hat{r}\right) = 0$$

After some simplification, Equation (4.6) results:

$$\sum_{i=1}^{n} v_i - n\hat{e} + n\mu\hat{r} = 0 \tag{4.6}$$

The other partial derivative gives

$$\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(\frac{v_i - \hat{e} + \mu\hat{r}}{r}\right)^2$$

$$\frac{\partial\big(\log\big(P(\boldsymbol{V}|\boldsymbol{\theta})\big)\big)}{\partial\hat{r}} = -\frac{n}{\hat{r}} - \frac{1}{\sigma^2}\sum_{i=1}^{n}\left(\frac{v_i - \hat{e}}{\hat{r}} + \mu\right)\left(-\frac{v_i - \hat{e}}{\hat{r}^2}\right) = 0$$

$$-\frac{n}{\hat{r}} - \frac{1}{\sigma^2}\sum_{i=1}^{n}\left(-\frac{(v_i - \hat{e})^2}{\hat{r}^3} - \mu\frac{(v_i - \hat{e})}{\hat{r}^2}\right) = 0$$

$$-n + \frac{1}{\hat{r}^2\sigma^2}\sum_{i=1}^{n}(v_i - \hat{e})^2 + \frac{\mu}{\hat{r}\sigma^2}\sum_{i=1}^{n}(v_i - \hat{e}) = 0$$

and hence the following nonlinear relation is obtained:

$$\hat{r}^2 - \frac{1}{n\sigma^2}\sum_{i=1}^{n}(v_i - \hat{e})^2 - \frac{\mu\hat{r}}{n\sigma^2}\sum_{i=1}^{n}(v_i - \hat{e}) = 0 \tag{4.7}$$

In Equation (4.6) and (4.7), n is the number of samples, σ is the variance of the current, μ is the mean of the current and $\sum_{i=1}^{n} v_i$ is the sum of the voltage samples. To obtain $\hat{e}$ and $\hat{r}$ we use the Newton's method. In the context of this numerical method $\hat{e}$ and $\hat{r}$ are the critical points of the system of equations formed by (4.6) and (4.7). This system is thus

$$f_1(\hat{e}, \hat{r}) = f_1(\hat{\theta}) = 0$$
$$f_2(\hat{e}, \hat{r}) = f_2(\hat{\theta}) = 0$$

by defining a compact vector

$$\boldsymbol{F}(\widehat{\boldsymbol{\theta}}) = [f_1 \quad f_2]'$$

the Newton's method set up is then

$$\widehat{\boldsymbol{\theta}}_{k+1} = \widehat{\boldsymbol{\theta}}_k - \left[\frac{d\boldsymbol{F}}{d\boldsymbol{\theta}}\right]^{-1}_{\widehat{\boldsymbol{\theta}}=\widehat{\boldsymbol{\theta}}_k} \boldsymbol{F}(\widehat{\boldsymbol{\theta}}_k) \tag{4.8}$$

It is clear that (4.6) and (4.7) will need to be differentiated again with respect to $\hat{e}$ and $\hat{r}$. The Jacobian matrix will then become a Hessian within the mathematical and programming algorithm. The following steps give the iterative process:

Step 1: Set $k = 0$, a tolerance $\varepsilon > 0$ and give an initial guess $\widehat{\boldsymbol{\theta}}_0 = \widehat{\boldsymbol{\theta}}_o$

Step 2: Compute the Jacobean $\left[\frac{d\boldsymbol{F}}{d\boldsymbol{\theta}}\right]$

Step 3: Solve (7) for $\widehat{\boldsymbol{\theta}}_{k+1}$ using $\widehat{\boldsymbol{\theta}}_k$

Step 4: If $norm\left(\left[\frac{d\boldsymbol{F}}{d\boldsymbol{\theta}}\right]_{\widehat{\boldsymbol{\theta}}=\widehat{\boldsymbol{\theta}}_k}\right) < \varepsilon$ go back to Step 2 else go to next step

Step 5: Done, the solution is $\widehat{\boldsymbol{\theta}}_{k+1} = [\hat{e}_{ML} \quad \hat{r}_{ML}]'$

The MATLAB code for this algorithm is given in Appendix B. For the results in Section 4.5 the same generated data in Section 4.1 was used. This method offers a robust alternative and, as the results show, requires fewer samples to give an accurate estimate of the TE parameters.

4.4 Results

The results in Table 4.1 were acquired by applying both methods to the same set of generated data. The actual values of the parameters are $E = 10\ V$ and $R = 2\ \Omega$. The generated data for current had a mean value of $I_o = 1\ A$ and hence the voltage has a mean of $V_o = 8\ V$. The results show the variance of the estimated parameters as the samples are increased.

Table 4.1 Results comparing LSE vs. MLE methods

	Actual	$\boldsymbol{n = 30}$	$\boldsymbol{n = 100}$	$\boldsymbol{n = 1000}$	$\boldsymbol{n = 10^4}$	$\boldsymbol{n = 10^5}$	$\boldsymbol{n = 10^6}$
$\hat{\boldsymbol{e}}$ **(LSE)**	10	8.555	9.961	9.704	9.967	10.001	9.999
$\hat{\boldsymbol{e}}$ **(MLE)**	10	10.929	9.986	10.032	10.167	10.167	10.150
$\hat{\boldsymbol{r}}$ **(LSE)**	2	0.031	2.283	1.930	1.963	1.989	2.003
$\hat{\boldsymbol{r}}$ **(MLE)**	2	2.386	2.222	2.152	2.089	2.151	2.155
$\boldsymbol{V_{mean}}$	8	8.542	7.764	7.879	8.078	8.016	7.995

CHAPTER 5

APPLICATION AND CONCLUSION

5.1 Application and Challenges of an ac System

We can apply LSE to the ac circuit, shown in Figure 1.3, as described in Chapter 3. Two of the four TEPs in the ac circuit can then be found ($Z = R + jX$) as seen in Figures 5.1 and 5.2. However, when computing the other two (in phasor voltage $E\angle\theta$) the results yield a scatter display of semi-random estimated values. This effect is due to the drift caused by the PMU as described in Appendix A.2.

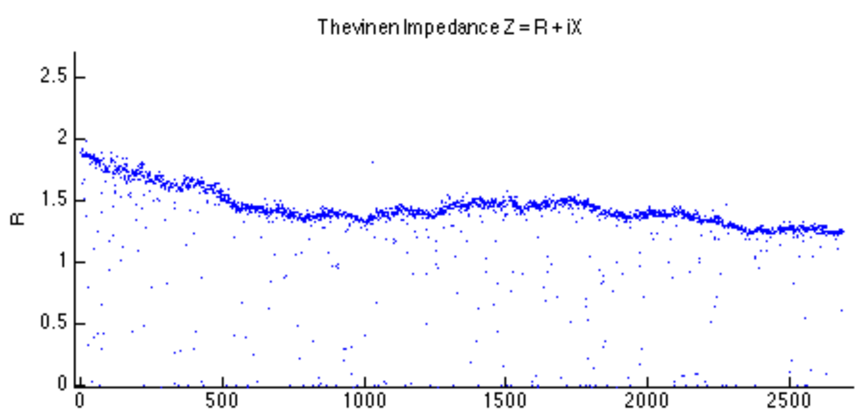

Figure 5.1 Estimated values of R. Part of the TEPs for the second PMU data set.

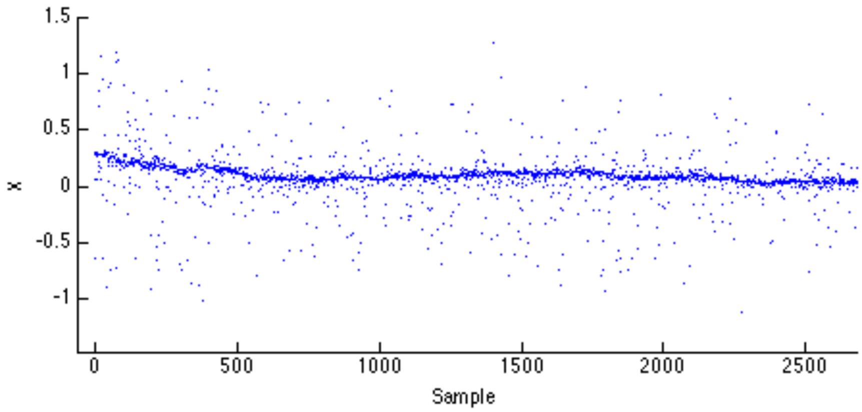

Figure 5.2 Estimated values of X. Part of the TEPs for the second PMU data set.

In order to avoid the drift effect, the following assumptions and definitions can be used:

- Set E_a and E_b to the base line voltage and assume $R \approx 0$ so that $Z = Xe^{j\frac{\pi}{2}}$, thus effectively reducing the set of unknowns to two (i.e. θ *and* X in station a and b).
- Change LSE using complex equations and Figure 3.1 as specified below:

$$\bar{E} = \bar{I}Z + V$$

$$|E|\cos\delta + X|I|\sin\theta_I = |V|\cos\theta_V$$

$$|E|\sin\delta - X|I|\cos\theta_I = |V|\sin\theta_V$$

  where $|E|$ is the base voltage, $|I|, |V|, \theta_I, \theta_V$ are the given measurements. Thus δ and X are the unknowns. The above system is solvable since there are two equations and two unknowns.
- Use reduced system as in Figure 1.3 with the difference of TE voltage phasor and define $\delta = \theta_a - \theta_b$. The drift then will be subtracted from both ends.

The latter point gives light to an application using the estimated TEP, in particular δ. In [24] loadability of a power system is analyzed based on the angle difference between two TE systems across a TL. This application can be very useful for monitoring the health of a TL, possible stability issues, and given the fast availability of data from PMU it can be done in real-time.

5.2 Conclusion

We have explored two different methods of estimating the TEP of a dc system based on the output measurements. This system was drawn from the original full system in Figure 1.1 and the measurements were those of the PMU data shown in Chapter 2.

We have used this simplified system for a specific example on the computational challenges by using LSE in Chapter 3. The minimum variation measurements must have, among each other, in order to avoid an ill-conditioned matrix, was found. Then a number of studies were performed to understand how to generate data, which avoids this conditionality. It was found that only by varying simulated load measurements could linearly independent samples be generated. In a complete complex power system such as the IEEE 30-bus system, the load in bus 30 is normally varied in order to simulate PMU data [2] and [5]. This process is consistent with the findings of the studies in Chapter 3. Finally in Chapter 4 the same process of estimation based on the noise in measurements was discussed. A comparison of MLE against LSE was given. This estimation technique is much harder to apply to a complex domain system and LSE is favored in such cases.

APPENDIX A
ABOUT THE PMU

A.1 Definition

Phasor measurement units (PMU), or synchrophasors as they are known within the industry, are precise grid monitors, which measure voltage, current and frequency at high sampling rates. The measurements are stored as phasors with magnitude and angle. Each measurement is time-stamped using a GPS signal. This will assure that measurements taken across a transmission line at different nodes, several miles apart, are taken at the exact same time for the same system state. The GPS signal contains the date, time with microseconds resolution, time zone and daylight time saving information. PMU measurements offer an accurate and faster analysis of the health of the transmission line they monitor than the classic SCADA system. This gives transmission system operators superior system visibility and reliability.

A. 2 Technical Specifications

PMU uses a transducer that converts the 3-phase sinusoidal voltage or current into phasors

$$x(t) = X_m \cos(wt + \theta) \quad \rightarrow \quad \bar{X} = \frac{X_m}{\sqrt{2}} \angle\theta$$

It does so using a recursive algorithm in which the phasor is converted from the measured signal using the Fourier series approximation:

$$\bar{X} = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x_k e^{-jk\frac{2\pi}{N}}$$

where N is the number of samples taken over one period (i.e. 1/60 s). The trigger signal for sampling is taken from a Global Position System (GPS), which is synchronized by atomic clocks. This GPS signal uses twenty-four satellites (with minimum visibility of a least 5 at any time anywhere) for precise position and timing. This ensures PMU measurements taken over a wide area, spanning hundreds of miles, are synchronized.

However, one drawback of the original PMUs was the assumed frequency for phasor conversion. At every measurement PMU uses the nominal frequency (60 Hz) for conversion. We know this is not true, and as a matter of fact the frequency continuously varies over time due to various reasons. Figure A.1 shows the variation of the frequency for the data set 1 over a one-hour period. As a result, phase angles are affected at off-nominal frequencies, creating a drift effect, which is the difference between the nominal and the system frequency. This drift is maintained and spread throughout the measurements for the period being monitored. Drift compensation is used in modern PMUs using special tracking algorithms. For more details on PMUs see [25].

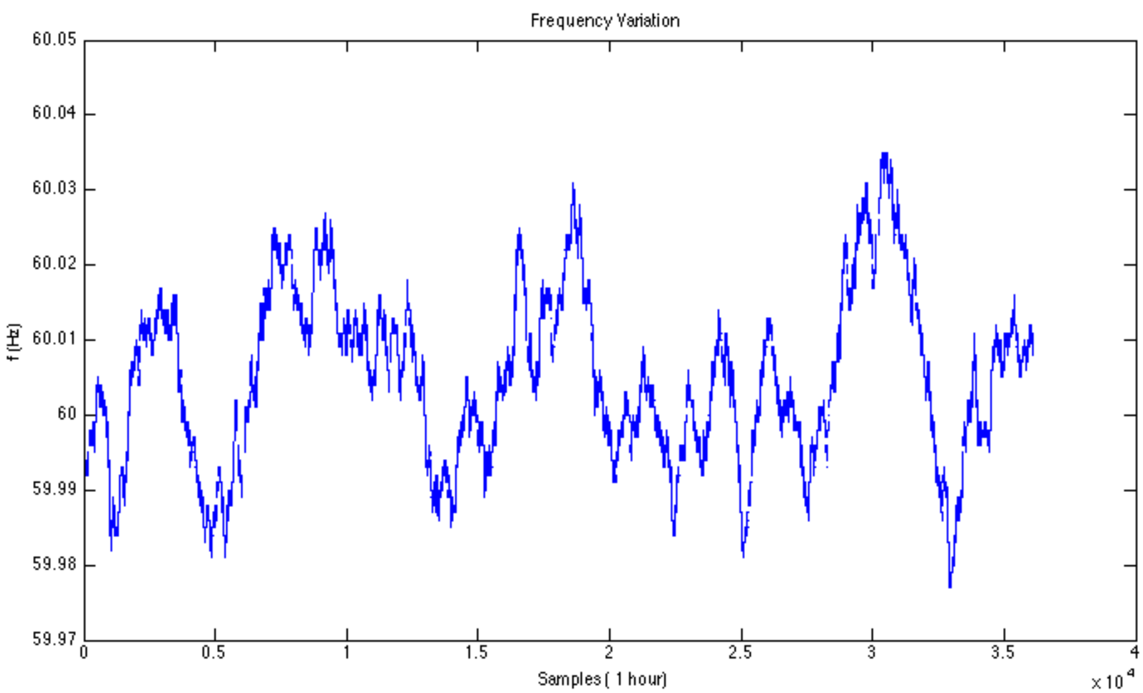

Figure A.1 System frequency variation over a one-hour period

APPENDIX B

MATLAB SIMULATION CODE

```
%Maximum Likelihood
%ECE 599
%This code estimates the parameters e (power source) and
%r (resistance) of a Thévenin equivalent system
%given the measured distribution of the output voltage V
%and current I

clc
clear

%The unknown parameters \theta
syms e r

%Generate measured distribution of I~N(u,s^2)
u = 1;
s = 2;
n=30;  %Number of samples

I = u + s*randn(n,1);

%Assume values of e,r to generate V distribution
E = 10;
R = 2;

E1= E+s.*randn(n,1);
R1= R+s.*randn(n,1);

%Observations based on the above
V=(E1-R1.*I);

% V = E-R*I;
V2 = V.^2;

%Theta parameters
t = [e;r];

%d(log F(V|t) )/dt
f1 = sum(V)-n*e+n*u*r;
f2 = -n*s^2*r^2+1/r*(sum(V2)-2*e*sum(V)+n*e^2)+u*(sum(V)-n*e);

f = [f1;f2];

J = jacobian(f,t); %Find roots of this eq
```

```
t_ml = [1;1];

for i=1:200
    f_eval = subs(f,t,t_ml);

    J_eval = subs(J,t,t_ml);

    vsol = t_ml(1)-t_ml(2)*u;

    if(norm(f_eval)<0.0001)
        break
    end

    t_ml = t_ml - J_eval\f_eval;

end

v_mean = mean(V);

display (vsol)
display (v_mean)
display (t_ml);
%END OF ML

%BEGIN LSE CODE
%Least Squares estimate of E and R
%Increase n = Number of observations above for better results

M=[ones(n,1),-I];

LSE=(((M'*M)^-1)*M')*V;

display(LSE);
```

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